Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Wednesday 5 June 2019

Morning (Time: 2 hours) | Paper Reference **9MA0/01**

Mathematics

Advanced

Paper 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P58353A

1/1/1/1/1/1/C2/C2/

P 5 8 3 5 3 A 0 1 4 4

Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. $$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

Since $(x+3)$ is a factor: $x+3=0$

$x=-3$ is a root of $f(x)$ so $f(-3)=0$

sub in $x=-3$

1 $f(-3) = 3(-3)^3 + 2a(-3)^2 - 4(-3) + 5a$ → Solve algebraically to get a value for a.

$0 = 3(-27) + 2a(9) - 4(-3) + 5a$

$0 = -81 + 18a + 12 + 5a$

★ USE BIDMAS!!

1 $+69$ $0 = -69 + 23a$ $+69$

$\div 23$ $69 = 23a$ $\div 23$

$3 = a$

1

→ indices first!
– then collect like terms

Question 1 continued

(Total for Question 1 is 3 marks)

2.

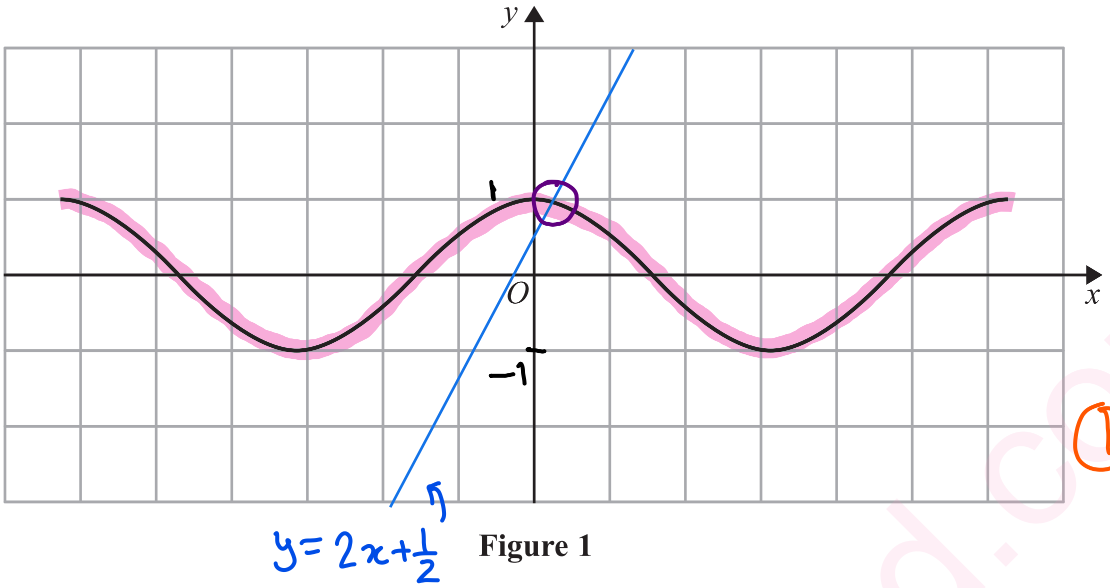


Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians. Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

Given that the root of the equation is α, and that α is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

a) $\cos x - 2x - \frac{1}{2} = 0$

→ Rearrange to get two equations:
$y = \cos x$ and $y = 2x + \frac{1}{2}$
then set them both equal to eachother to find their point of intersection.

$\cos x = 2x + \frac{1}{2}$

$y = 2x + \frac{1}{2}$ → y intercept

gradient

1

Conclude → There is only one real solution because $y = \cos x$ and $y = 2x + \frac{1}{2}$ only intersect at one point.
(Solution is circled on the graph)

Question 2 continued

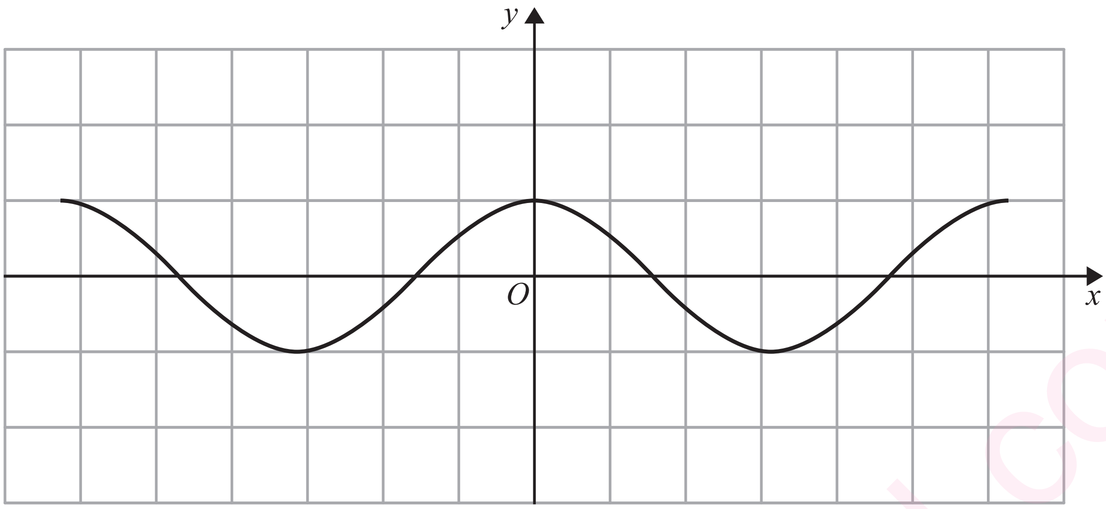


Diagram 1

b) For small angles $\cos\theta \approx 1-\frac{\theta^2}{2}$

Given in the formula booklet:
Small angle approximation
$\sin\theta \approx \theta$
$\cos\theta \approx 1-\frac{\theta^2}{2}$
$\tan\theta \approx \theta$

$\theta = x$

$\cos x - 2x - \frac{1}{2} = 0$ (1)

$1-\frac{x^2}{2} - 2x - \frac{1}{2} = 0$

×2 to get rid of fractions

$2 - x^2 - 4x - 1 = 0$

×−1 $-x^2 - 4x + 1 = 0$

$x^2 + 4x - 1 = 0$ (1)

Learn this

↳ Put this in your calc or use quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Solve for x:

$x = \frac{-(4) \pm \sqrt{(4)^2-(4 \times 1 \times -1)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2}$

Reject negative values ← $x = \frac{-4-\sqrt{20}}{2} = -2-\sqrt{5} = -4.236\ldots$

(1)

$x = \frac{-4+\sqrt{20}}{2} = -2+\sqrt{5} = 0.236\ldots$

$x = -2+\sqrt{5} \rightarrow x = 0.236$ (3sf)

Only the positive solution is used because the intersection is in the first quadrant: all positive.

(Total for Question 2 is 5 marks)

3. $$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\frac{dy}{dx} = \frac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\frac{dy}{dx} < 0$

(1)

a) Since this is a division, we can use the Quotient rule or the chain rule. (we will use the Quotient rule)

Quotient rule: Given in the formula booklet:

$$\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

also written as $f'(x) = \frac{u'v - uv'}{v^2}$

$u = 5x^2 + 10x \xrightarrow{\frac{dy}{dx}} u' = 10x + 10$

★ simple differentiation: $y = x^n \longrightarrow \frac{dy}{dx} = nx^{n-1}$

$v = (x+1)^2 = x^2 + 2x + 1 \xrightarrow{\frac{dy}{dx}} v' = 2x + 2$ (1)

expand out the brackets

Substitute in u, u', v, v' back into our formula:

$$f'(x) = \frac{[(10x+10)(x+1)^2] - [(5x^2+10x)(2x+2)]}{[(x+1)^2]^2}$$ (1)

factorise $(2x+2)$

$$f'(x) = \frac{[(10x+10)(x+1)\cancel{(x+1)}] - [(5x^2+10x) \times 2\cancel{(x+1)}]}{(x+1)(x+1)(x+1)\cancel{(x+1)}}$$

cancel out $(x+1)$

$$f'(x) = \frac{[(10x-10)(x+1)] - [(5x^2+10x) \times 2]}{(x+1)(x+1)(x+1)}$$ (1)

expand brackets then simplify.

$$f'(x) = \frac{[10x^2 - 10] - [10x^2 + 20]}{(x+1)^3}$$

$$f'(x) = \frac{10}{(x+3)^3}$$ (1)

$A = 10$

$n = 3$

DO NOT WRITE IN THIS AREA

Question 3 continued

b) $\frac{10}{(x+1)^3} < 0$

↳ for $\frac{10}{(x+1)^3}$ to be negative, the denominator must be negative [as 10 can't be changed]

$(x+1)^3 < 0$

↳ negative

$(x+1) < 0$ ⟶ because any number squared is positive ⟶ positive × negative = negative.

$x = -1$ #

1

(Total for Question 3 is 5 marks)

4. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to $\sqrt{2}$
Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$
- $x = 2$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$
- $x = -\dfrac{1}{2}$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{\dfrac{9}{2}}} = \dfrac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used

(1)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$

(1)

a) $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{(4-x)^{\frac{1}{2}}} = (4-x)^{-\frac{1}{2}}$ → Substitute this into binomial formula

$\hookrightarrow (4-x)^{-\frac{1}{2}} = \left[4\left(1-\frac{x}{4}\right)\right]^{-\frac{1}{2}} = \sqrt{\frac{1}{4}}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ 1

Given in the formula booklet:

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{1\times 2}x^2 + \dots + \dfrac{n(n-1)\dots(n-r+1)}{1\times 2\times\dots\times r} + \dots$ $(|x|<1, n\in\mathbb{R})$

$n = -\dfrac{1}{2}$ $\quad x = \dfrac{x}{4}$ 1

$\frac{1}{2}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = \frac{1}{2}\left[1 + \left[\left(-\frac{1}{2}\right)\left(\frac{-x}{4}\right)\right] + \left[\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-\frac{2}{2}\right)}{2}\left(\frac{-x}{4}\right)^2\right] + \dots\right]$

Simplifying

$\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2} = \dfrac{\frac{3}{2}}{2}$

$= \frac{3}{2}\times\frac{1}{2} = \frac{3}{4}$

$= \frac{3}{4}\times\left(\frac{x}{4}\right)^2$

$= \frac{3}{4}\times\frac{x^2}{16} = \frac{3x^2}{128}$

$\frac{1}{2}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = \frac{1}{2}\left[1 + \frac{x}{8} + \frac{3x^2}{128}\right] + \dots$ 1

$\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{2} + \dfrac{x}{16} + \dfrac{3x^2}{256} + \dots$ 1

DO NOT WRITE IN THIS AREA

Question 4 continued

from the formula

b) i) $\frac{1}{2}\left(1+-\frac{x}{4}\right)^{-\frac{1}{2}}$ $(|x|<1)$ $\xrightarrow{\text{Sub in } x=\frac{-x}{4}}$ $\left|\frac{-x}{4}\right|<1$

$= |-x| < 4$

$= |x| < 4$ (1)

ii) Conclusion → Since $|-14|=14$ and $14>4$, and that does not meet the $|x|<4$ requirement, $x=-14$ should not be used. (1)

(Total for Question 4 is 6 marks)

5. $$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write f(x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

a) $f(x) = 2x^2 + 4x + 9 \quad x \in \mathbb{R}$

$f(x) = 2(x^2 + 2x) + 9 \leftarrow$ Completing the square 1

$f(x) = 2(x+1)^2 - 2 + 9$ 1

$f(x) = 2(x+1)^2 + 7$ 1

b) ① y intercept = 9

② x intercept $\rightarrow y = 0$

$0 = 2(x+1)^2 + 7$

$2(x+1)^2 = -7$

$(x+1)^2 = \frac{-7}{2} \leftarrow$ No real roots

$x + 1 = \sqrt{\frac{-7}{2}} \rightarrow$ gives imaginary numer $\sqrt{-1}$

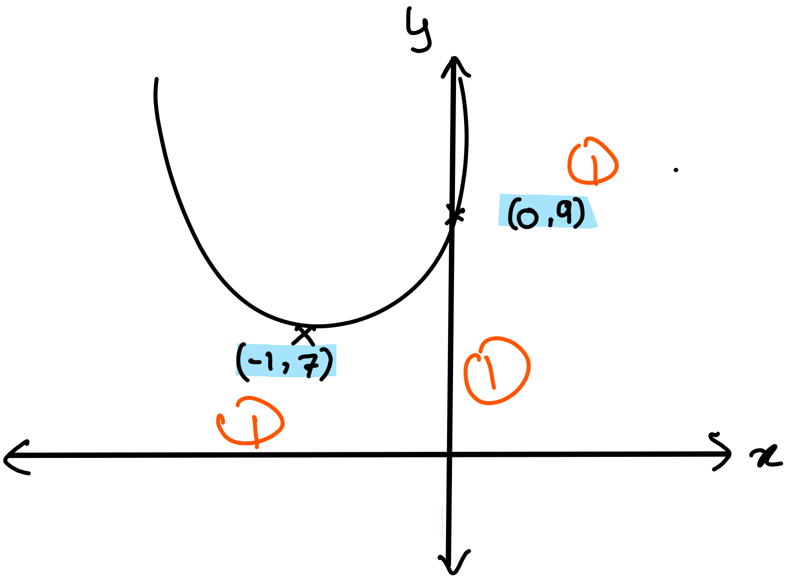
③ turning point from part a

$f(x) = 2(x+1)^2 + 7$

$x+1 \downarrow \quad \downarrow$

$(-1, 7)$

Question 5 continued



c) i) $g(x) = 2(x-2)^2 + 4x - 3 \quad x \in \mathbb{R}$.

$f(x) = 2x^2 + 4x + 9$ $\quad f(x-a) \longrightarrow \begin{pmatrix} a \\ 0 \end{pmatrix}$ translation

$f(x-2) = 2(x-2)^2 + 4(x-2) + 9$ $\quad f(x) + b \longrightarrow \begin{pmatrix} 0 \\ b \end{pmatrix}$ translation

$f(x-2) = 2(x-2)^2 + 4x - 8 + 9$

$f(x-2) = 2(x-2)^2 + 4x + 1$

$f(x-2) = 2(x-2)^2 + 4x + 1$ $\quad -4$ [difference of 4 so $g(x) = f(x-2) - 4$]

$g(x) = 2(x-2)^2 + 4x - 3$

$-a = -2 \quad a = 2 \quad \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

$g(x) = f(x-2) - 4 \longrightarrow b = -4$ $\quad$ (2)

f(x) is transformed right by 2, and down by 4. Overall transformation is a translation by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ to get g(x).

Question 5 continued

c) ii) $h(x) = \dfrac{21}{2x^2+4x+9}$ $x \in \mathbb{R}$

$h(x) = \dfrac{21}{f(x)}$ = $f(x) \Rightarrow \pm\infty$ (max value)

$h(x) \Rightarrow 0 \leftarrow \dfrac{21}{\infty}$

$f(x) \Rightarrow 7$ (min value) ①

$h(x) \Rightarrow 3 \leftarrow \dfrac{21}{7}$

$0 < h(x) \leqslant 3$ ①

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

P 5 8 3 5 3 A 0 1 2 4 4

Question 5 continued

(Total for Question 5 is 10 marks)

6. (a) Solve, for $-180° \leqslant \theta \leqslant 180°$, the equation

$$5\sin 2\theta = 9\tan\theta$$ → Algebraic manipulation

giving your answers, where necessary, to one decimal place.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(b) Deduce the smallest positive solution to the equation

$$5\sin(2x - 50°) = 9\tan(x - 25°)$$

(2)

a) $5\sin 2\theta = 9\tan\theta$ trig identity: $\sin 2\theta = 2\sin\theta\cos\theta$

$5(2\sin\theta\cos\theta) = 9\tan\theta$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$10\sin\theta\cos\theta = 9\dfrac{\sin\theta}{\cos\theta}$ (1)

$10\sin\theta\cos^2\theta = 9\sin\theta$ $\times\cos\theta$

$10\sin\theta\cos^2\theta - 9\sin\theta = 0$

Factor out $\sin\theta$

$\sin\theta(10\cos^2\theta - 9) = 0$

either $\sin\theta = 0$ or $10\cos^2\theta - 9 = 0$ (1)

$\sin^{-1}(0) = \theta$ use calc

$= 0°$

$10\cos^2\theta = 9$

$\cos^2\theta = \dfrac{9}{10}$

$\cos\theta = \pm\sqrt{\dfrac{9}{10}}$

$\cos^{-1}\left(\dfrac{\pm 3}{\sqrt{10}}\right)$ use calc

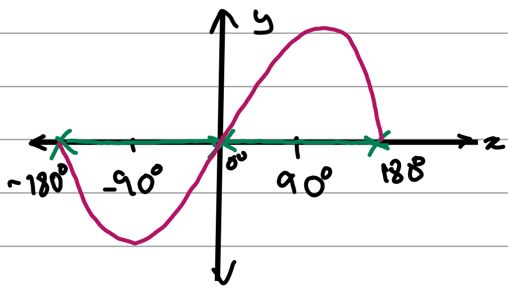
(1)

$\theta = \cos^{-1}\left(\dfrac{3}{\sqrt{10}}\right) = 18.4°$ $\cos^{-1}\left(\dfrac{-3}{\sqrt{10}}\right) = 161.6°$

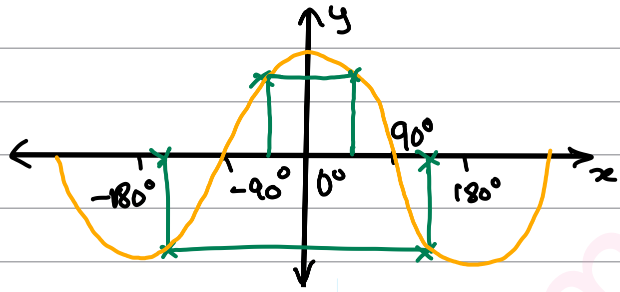
These are only the principle value so we nee to find more θ value that fall in the range: So we will use 1 graph method.

Question 6 continued

$y = \sin(x)$



$y = \cos(x)$



$\theta = 0°, -180°, 180°$

$\theta = 161.6°, -161.6°, 18.4°, -18.4$

both solutions ($\sin\theta$ & $\cos\theta$)

$\theta = 0°, 180°, -180°, 161.6°, -161.6°, 18.4°, -18.4°$ (3)

b) $\theta = x - 25°$

$25 + \theta = x$ $\quad \theta = -18.4°$ (1)

$25 - 18.4 = x$

$6.6° = x$ (1)

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000 → new so $t=0$
- its value after one year is £16 000 → $t=1$

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

a) $V = Ae^{kt}$ → similar to $y=e^x$ model (1)

At $t=0$

$V = 20{,}000 = Ae^{k(0)}$

$20{,}000 = Ae^0$ → $n^0 = 1$

$\underline{A = 20{,}000}$ (1)

$V = 20{,}000e^{kt}$

At $t=1$

$V = 16000 = 20{,}000e^{k(1)}$

$16000 = 20000e^k$ (1)

$e^k = \frac{16000}{20000} = \frac{16}{20} = \frac{4}{5}$

$\ln(e^k) = \ln\left(\frac{4}{5}\right)$ → use ln to get rid of e^k

$k = \ln\left(\frac{4}{5}\right)$

$k = -0.22314\ldots = -0.223$ (3sf)

$V = Ae^{kt}$ → $\underline{V = 20{,}000e^{-0.223t}}$ (1)

Question 7 continued

b) $t = 10$

$V = 20000e^{-0.223(10)}$

$V = £2150$ 1

Our model is reliable because £2150 is only £150 off the actual value of the car after 10 years. 1

c) Since car B has the same value as A when new our A value in the $V = Ae^{kt}$ equation will remain the same.
But since the value of the car depreciates more slowly, k would have to be made smaller than -0.223. 1

P 5 8 3 5 3 A 0 1 9 4 4

Question 7 continued

Question 7 continued

(Total for Question 7 is 7 marks)

8.

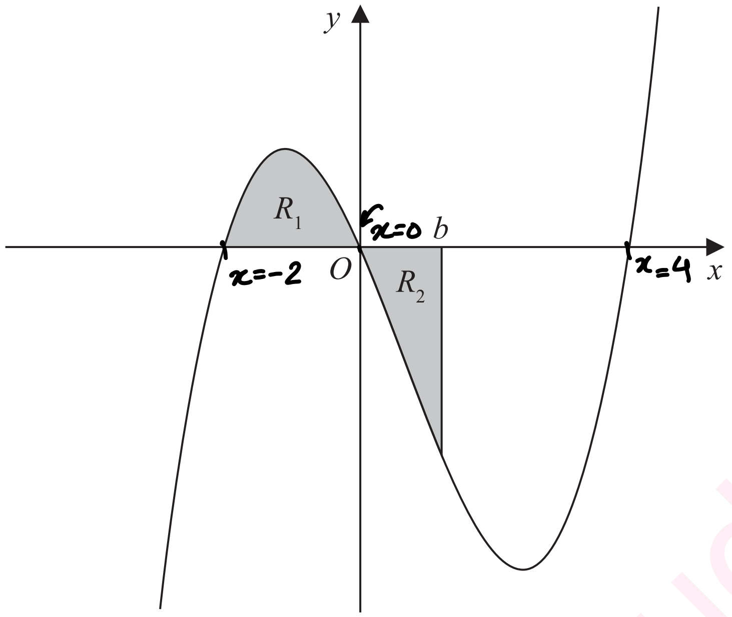


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2 $\rightarrow \frac{20}{3}$

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places.
The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

a) $y = x(x+2)(x-4) \longrightarrow$ solution: $x = 0$
$x = -2$
$x = 4$

1

$\int_{-2}^{0} x(x+2)(x-4) = \int_{-2}^{0} x^3 - 2x^2 - 8x \, dx$ → Intigrate to find the area between curve and x-axis

expand fully before intigrating.

DO NOT WRITE IN THIS AREA

Question 8 continued

$\int_{-2}^{0} x^3 - 2x^2 - 8x \, dx$

$= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0}$ (1)

$= (0) - \left(\frac{1}{4}(-2)^4 - \frac{2}{3}(-2)^3 - 4(-2)^2\right)$ ← Sub in 0 & -2 limits

(1)

$= 0 - \left(-\frac{20}{3}\right)$

no +C needed because they will cancel out when limits are applied.

$= \frac{20}{3}$ ← as required (1)

b) $\int_{0}^{b} x^3 - 2x^2 - 8x \, dx = -\frac{20}{3}$ ← from Q

rearrange and simplify

$= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{0}^{b} = -\frac{20}{3}$ (1)

$= \frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 - 0 = -\frac{20}{3}$

$1b^4 - \frac{8}{3}b^3 - 16b^2 - 0 = -\frac{80}{3}$

$3b^4 - 8b^3 - 48b^2 = -80$

$3b^4 - 8b^3 - 48b^2 + 80 = 0$ (1)

$(b+2)^2(3b^2 - 20b + 20) = 0$

$(b^2 + 4b + 4)(3b^2 - 20b + 20) = 0$ (1)

$3b^4 - 20b^3 + 20b^2 + 12b^3 - 80b^2 + \cancel{80b} + 12b^2 - \cancel{80b} + 80 = 0$

$3b^4 - 8b^3 - 48b^2 + 80 = 0$ (1)

Both equations are equal so we have correctly verfied that b satisfies the equation.

Question 8 continued

c)

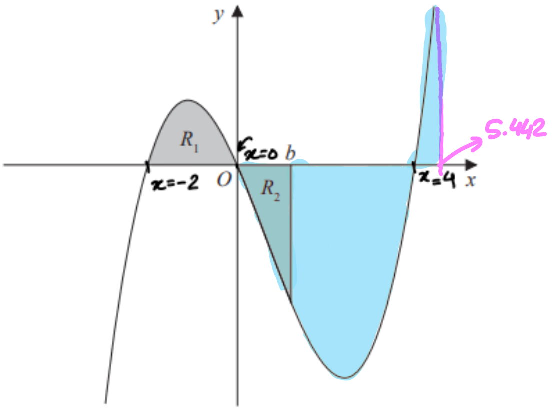


Figure 2

①

Since $R_1 = R_2$ $\int_0^b y\,dx = -\frac{20}{3}$ b= 1.225 and 5.442

↳ from the q.

$$\int_0^{1.225} y\,dx = -\frac{20}{3} \longrightarrow \int_0^{5.442} y\,dx = -\frac{20}{3}$$

The net area between 0 and 5.442 is the same as the area between 0 an 1.225 becaus $\int_4^6 y\,dx$ and $\int_4^{5.442}$ cancel out

↳ $\int_0^{1.225} y\,dx = \frac{20}{3}$ ①

$\int_{1.225}^{5.442} y\,dx = 0$ ← the net area is 0 as they cancel out.

$\int_0^{5.442} y\,dx = (-)\frac{20}{3}$

Question 8 continued

(Total for Question 8 is 10 marks)

9. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b-1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

a) $\log a - \log b = \log(a-b)$ Use laws of logarithm 1

$\log\left(\frac{a}{b}\right) = \log(a-b)$

$\log_x(a) + \log_x(b) = \log_x(ab)$

$\frac{a}{b} = a-b$

$\log_x(a) - \log_x(b) = \log_x\left[\frac{a}{b}\right]$

$a = ab - b^2$ Simplify and rearrange

$b^2 = ab - a$ 1

$b^2 = a(b-1)$

$a = \frac{b^2}{b-1}$ ← as required. 1

b) $a = \frac{b^2}{b-1}$ → the denominator can't be 0 $b-1 \neq 0$ so $b \neq 1$
→ and since b^2 is always positive and a must be positive, the denominator must also be positive so $b-1>0$ 1

$b-1>0$ 1

Question 9 continued

(Total for Question 9 is 5 marks)

10. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

$n^2+2 \longrightarrow$ statement

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."
State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

i) Since $n \in \mathbb{N} \rightarrow n$ can either be even or odd.

$\rightarrow$ When n is even let $n = 2k$

$n^2 + 2 = (2k)^2 + 2$

$= 4k^2 + 2 \longrightarrow$ not divisible by 4

(1)

So the statement isn't true for even values of n.

$\rightarrow$ When n is odd let $n = 2k+1$

$n^2 + 2 = (2k+1)^2 + 2$

$= 4k^2 + 4k + 1 + 2$

(1)

$= 4k^2 + 4k + 3$

$= 4(k^2 + k) + 3 \longrightarrow$ Not divisible by 4

So the statement is also not true for odd values of n.

(1)

Conclusion

Since n^2+2 isn't divisible by 4 for both even integers and for odd integers, n^2+2 isn't divisible by 4 for all $n \in \mathbb{N}$.

(1)

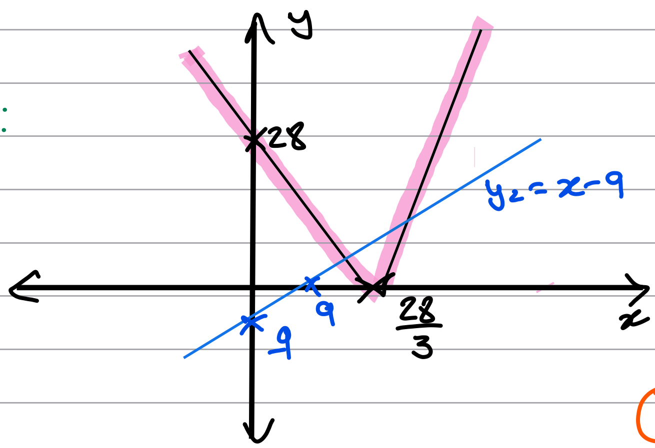
ii) $|3x - 28| \geq (x - 9)$

Draw the graph of both:

$y = |3x - 28|$ & $y = x - 9$

$y = 3x - 28$

$y = -3x + 28$



(1)

(1)

The statement is sometimes true because there are points where $y = x - 9$ is greater than $|3x - 28|$ but it is also smaller at some points.

Question 10 continued

(Total for Question 10 is 6 marks)

11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre. After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds, **(2)**

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leqslant r \leqslant 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race. **(4)**

a) Total time $= (6\times 4) + (6\times 1.05) + (6\times 1.05^2)$ $\rightarrow 24$

$= 36.915 \longrightarrow$ 36 minutes 55 seconds $\rightarrow$ as required

2

b) 5^{th} km $\rightarrow 6\times 1.05^{5-4} = 6\times 1.05$

6^{th} km $\rightarrow 6\times 1.05^{6-4} = 6\times 1.05^2$

7^{th} km $\rightarrow 6\times 1.05^{7-4} = 6\times 1.05^3$

Hence the time for the r^{th} km is $6\times 1.05^{r-4}$ min

1

c) Total time = time for first 4km + time for the last 16km

time for first 4km $= 6\times 4 = 24$ min 1

time for the last 16km $= \sum_{r=5}^{r=20} (6\times 1.05^{r-4})$ min

Given in the Booklet: Sum of a Geometric sequence $S_n = \dfrac{a(1-r^n)}{1-r}$

1 total time $= 24 + \sum_{r=5}^{r=20} (6\times 1.05^{r-4})$

1 $= 24 + \dfrac{6.3(1-1.05^{16})}{1-1.05}$

$a = 6.3 \quad r = 1.05$

$n = 16$

Total time = 173 minutes and 3 seconds

Question 11 continued

Turn over

Question 11 continued

Question 11 continued

(Total for Question 11 is 7 marks)

12. $$\mathrm{f}(x) = 10e^{-0.25x}\sin x, \quad x \geqslant 0$$

(a) Show that the x coordinates of the turning points of the curve with equation $y = \mathrm{f}(x)$ satisfy the equation $\tan x = 4$

(4)

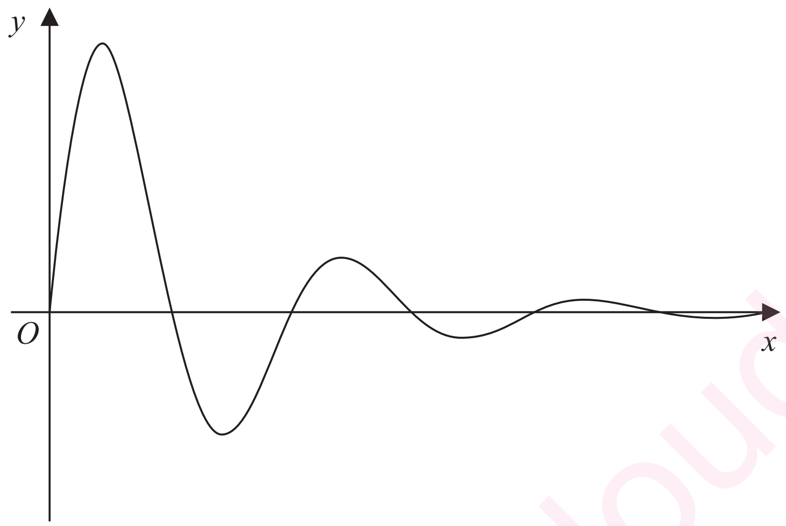


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$.

(b) Sketch the graph of H against t where

$$\mathrm{H}(t) = \left|10e^{-0.25t}\sin t\right| \qquad t \geqslant 0$$

showing the long-term behaviour of this curve. ↳ Let $f(x) = |10e^{-0.25t}\sin t|$

(2)

The function $\mathrm{H}(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

(3)

(d) Explain why this model should not be used to predict the time of each bounce.

(1)

a) $f(x) = 10e^{-0.25x}\sin x$ To diffentiate $y = ae^{xb}$ $\frac{dy}{dx} = abe^{xb}$

$f'(x) = -0.25(10e^{-0.25x})\sin x + \cos x(10e^{-0.25x})$

$= -2.5\sin x e^{-0.25} + 10\cos x e^{-0.25x}$

$\frac{dy}{dx} = 0$ for turning points 2

$0 = -2.5\sin x e^{-0.25x} + 10\cos x e^{-0.25x}$

$0 = e^{-0.25x}(-2.5\sin + 10\cos x)$ factor out $e^{-0.25x}$

Question 12 continued

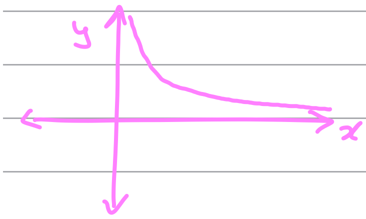
$0 = e^{-0.25x}(-2.5\sin x + 10\cos x)$

either $e^{-0.25x} = 0$ or $-2.5\sin x + 10\cos x = 0$

$e^{-0.25x} = 0$

has no solution

$x=0$ is an asymptote of $y = e^x$



$10\cos x = 2.5\sin x$

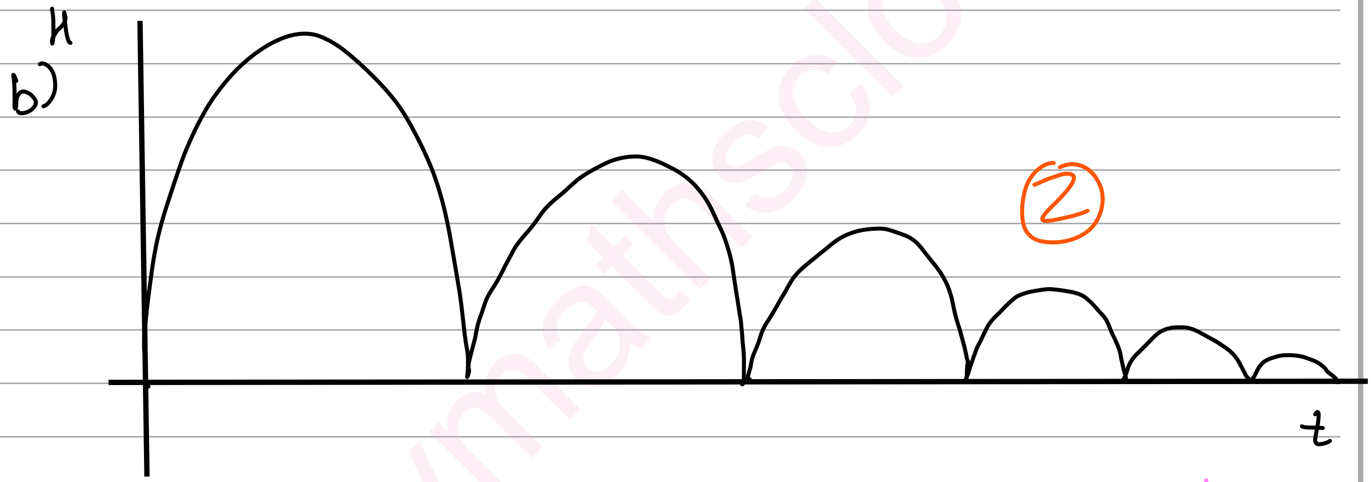
$\frac{10}{2.5} = \frac{\sin x}{\cos x}$

$4 = \tan x$

as required.

trig identity: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

b)



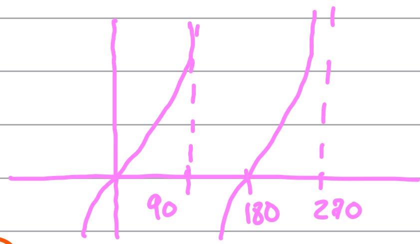
(2)

c) $\tan(x) = 4$

$x = \tan^{-1}(4)$ (1)

$= 1.326$ $+\pi$ (tan repeats every 180°)

$= 4.47$ (1)



(1)

$H(4.47) = \left|10e^{-0.25\times4.47}\sin 4.47\right| = |-3.175| = 3.18\text{m}$ (3sf)

d) The times between each bounce should not stay the same when the heights of each bounce is getting smaller.

(1)

Question 12 continued

Question 12 continued

(Total for Question 12 is 10 marks)

13. The curve C with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \qquad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where p and q are constants, passes through the point $\left(3, \frac{1}{2}\right)$ and has two vertical asymptotes with equations $x = 2$ and $x = -3$

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

(3)

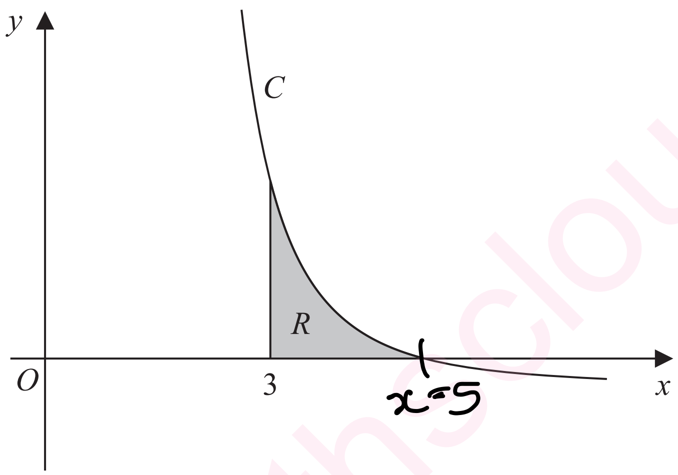


Figure 4

Figure 4 shows a sketch of part of the curve C. The region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(b) Show that the exact value of the area of R is $a \ln 2 + b \ln 3$, where a and b are rational constants to be found.

(8)

a) i) $(2x-q)(x+3)=0$

$2x-q=0 \qquad x+3=0$

$2(2)-q=0 \qquad x=-3$ ①

$4-q=0$

$q=4 \rightarrow$ as required

Sub in known coordinates to find p:

ii) $y = \dfrac{p-3x}{(2x-q)(x+3)} \rightarrow \left(3, \frac{1}{2}\right) \longrightarrow \frac{1}{2} = \dfrac{p-3(3)}{(2(3)-4)(3+3)}$ ①

$\frac{1}{2} = \frac{p-9}{12}$ ①

$6 = p-9$

$p = 6+9 = 15$ $\quad p=15$

P 5 8 3 5 3 A 0 3 8 4 4

Question 13 continued

b) finding the x intercept:

$$y = \frac{15-3x}{(2x-4)(x+3)}$$

$$0 = 15 - 3x$$
$$3x = 15$$
$$x = 5$$

Use partial fractions to split the fraction:

$$\frac{15-3x}{(2x-4)(x+3)} = \frac{A}{(2x-4)} + \frac{B}{(x+3)} \quad 2$$

$$15 - 3x = A(x+3) + B(2x-4)$$

Let $x = -3 \longrightarrow$ sets A as 0

$$15 - 3(-3) = B(2(-3)-4) + 0A$$
$$24 = -10B$$
$$-2.4 = B$$

Let $x = 2 \longrightarrow$ sets B as 0

$$15 - 3(2) = A(2+3) + 0B$$
$$9 = 5A$$
$$\frac{9}{5} = A = 1.8$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\frac{15-3x}{(2x-4)(x+3)} = \frac{1.8}{(2x-4)} - \frac{2.4}{(x+3)} \quad 1$$

$$\int_3^5 \frac{1.8}{(2x-4)} - \frac{2.4}{(x+3)}\,dx = \Big[0.9\ln|2x-4| - 2.4\ln|x+3|\Big]_3^5$$

$\downarrow$ $\qquad\qquad$ $\downarrow$

$f(x) = 2x-4$ $\qquad$ $f(x) = x+3$ $\qquad$ 4

$f'(x) = 2$ $\qquad$ $f(x) = 1$

$\frac{1.8}{2} = 0.9$ $\qquad$ $\frac{2.4}{1} = 2.4$

DO NOT WRITE IN THIS AREA

Question 13 continued Simplify :

$= 0.9\ln|2(5)-4| - 2.4\ln|5+3| - [0.9\ln|2(3)-4| - 2.4\ln|3+3|]$

$= 0.9\ln|6| - 2.4\ln|8| - 0.9\ln|2| + 2.4\ln|6|$

$= 3.3\ln|6| - 2.4\ln|8| - 0.9\ln|2|$

$= 3.3(\ln|3| + \ln|2|) - 2.4(\ln|2^3|) - 0.9\ln|2|$

$= 3.3(\ln|3|) + 3.3(\ln|2|) + 7.2(\ln|2|) - 0.9(\ln|2|)$

$= 3.3\ln|3| - 4.8\ln|2|$ // 1

ln laws are similar to log laws:

$\ln(ab) = \ln(a) + \ln(b)$

$\ln(a^b) = b(\ln(a))$

$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 13 continued

(Total for Question 13 is 11 marks)

14. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

a) $x = 4\sin(2y)$

if $x = \sin 2y \rightarrow \frac{dx}{dy} = 2\cos 2y$

$\frac{dx}{dy} = 4(2\cos(2y))$

$= 8\cos(2y)$ 1

$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{1}{8\cos 2y}$ reciprocal

$\frac{dy}{dx}$ at $0,0 \rightarrow \frac{1}{8\cos(0)} = \frac{1}{8}$ 2

b) i) $\sin x \approx x$ 1

↳ from small angle approximation

$\sin 2y = 2y$

$x = 4\sin 2y$

$x = 4(2y) \longrightarrow x = 8y$

ii) The value found in a is the gradient of the line found in b i. 1

DO NOT WRITE IN THIS AREA

Question 14 continued

c) $\frac{dy}{dx} = \frac{1}{8\cos 2y}$

Rearrange and Use trig identities:

$x = 4\sin 2y$

$\sin^2\theta + \cos^2\theta = 1$

$x^2 = 16\sin^2 2y$

$\sin^2\theta = 1 - \cos^2\theta$

$x^2 = 16(1 - \cos^2 2y)$

$x^2 = 16 - 16\cos^2 2y$ (1)

$16\cos^2 2y = 16 - x^2$

$\cos^2 2y = 1 - \frac{x^2}{16}$

$\cos 2y = \sqrt{1 - \frac{x^2}{16}}$ ← sub this back to $\frac{dy}{dx}$

(1)

$\frac{dy}{dx} = \frac{1}{8\sqrt{1 - \frac{x^2}{16}}}$

$\frac{dy}{dx} = \frac{\sqrt{16}}{8\sqrt{16 - x^2}}$

$\frac{dy}{dx} \; \frac{1}{2\sqrt{16 - x^2}}$ (1)

$a = 2 \quad b = 16$

P 5 8 3 5 3 A 0 4 3 4 4

Question 14 continued

(Total for Question 14 is 7 marks)

TOTAL FOR PAPER IS 100 MARKS